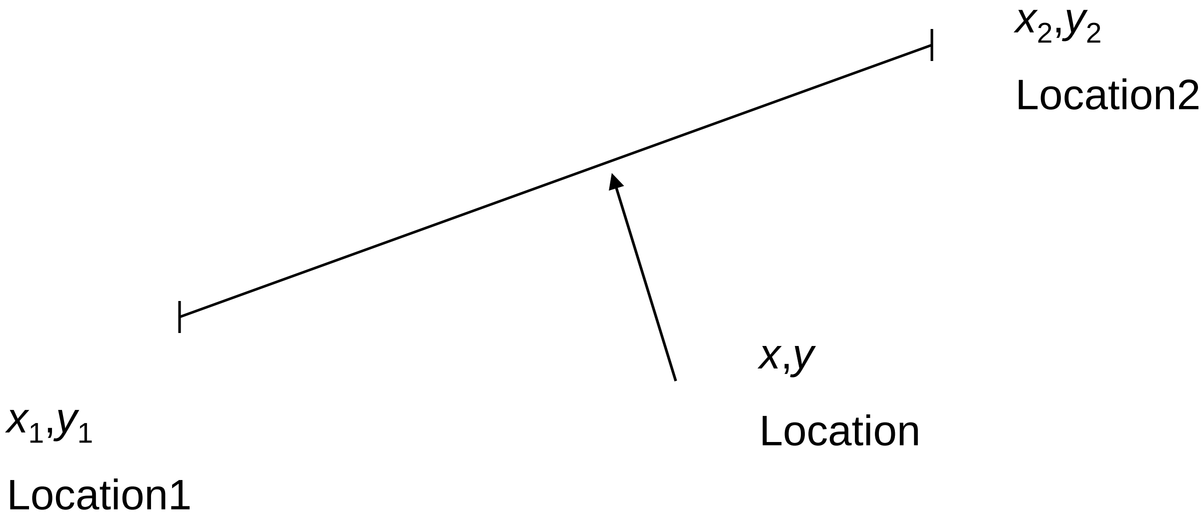
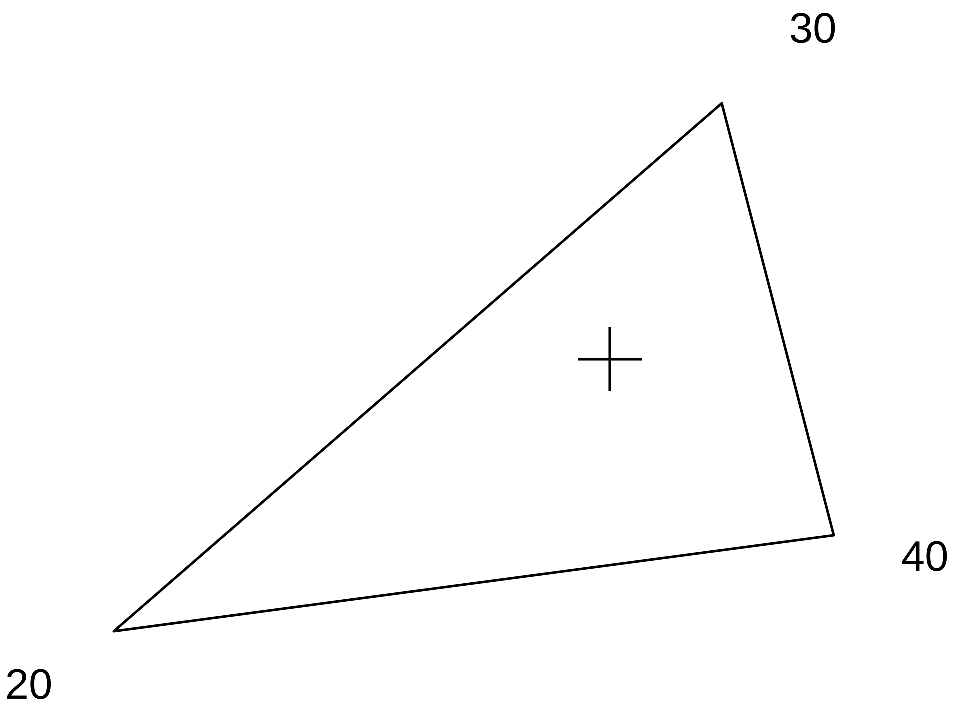
# Interpolation

# Linear interpolation

- Along a line
  - geocoding with coordinates

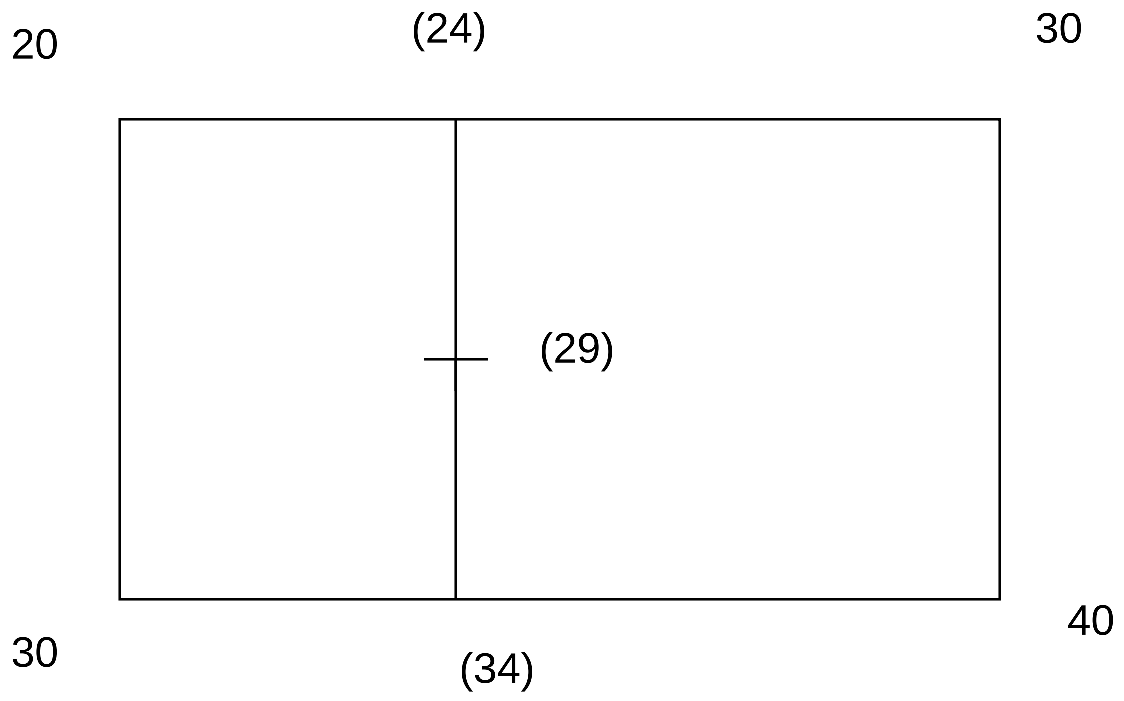


# In a triangle



# In a rectangle

- Bilinear interpolation



# Spatial interpolation

- There are two ways to represent continuous surface : one is a regular or gridded form, and the other is an irregular form
  - Regular: control points to gridded surface
  - Irregular: control points to triangulated surface

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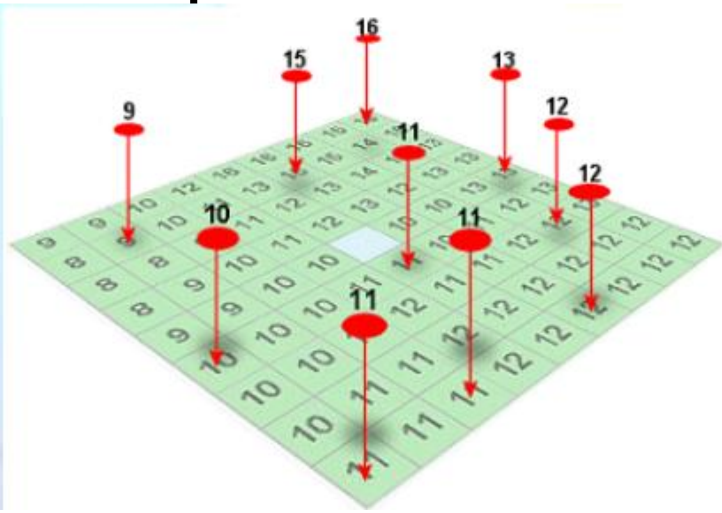
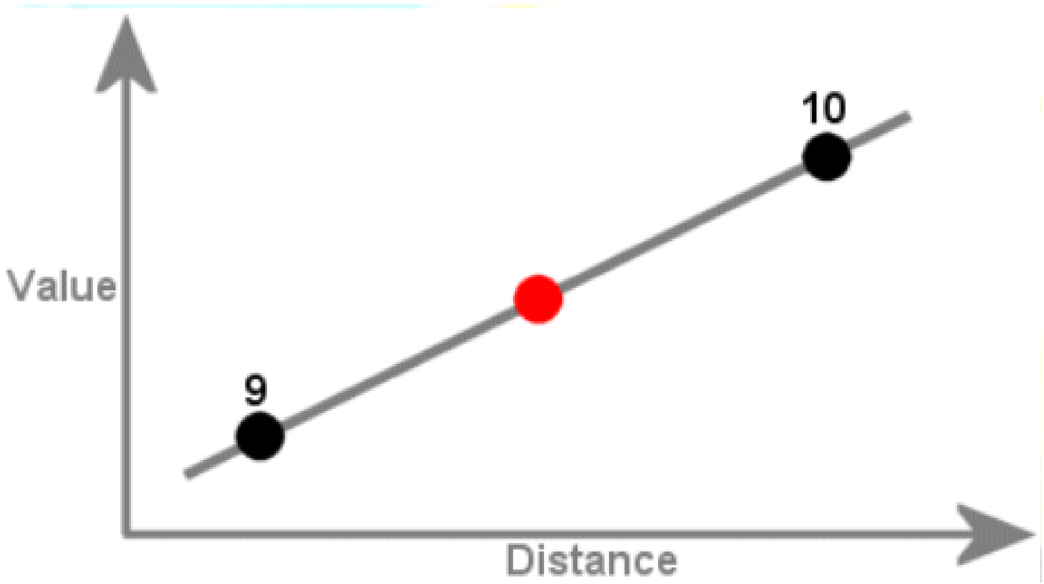
- The primary assumption of spatial interpolation is that points near each other are more al er away; therefore, any location's valu ated based on the values of points near



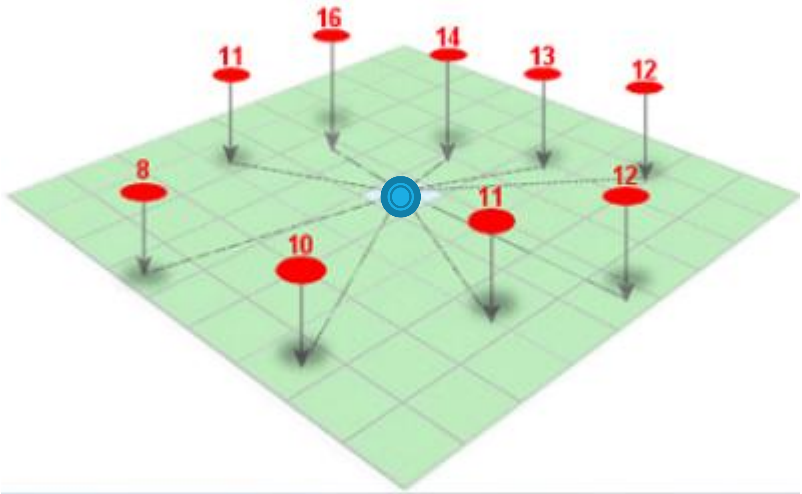
# What is interpolation?

- Interpolation is the process of estimating unknown values that fall between known values.



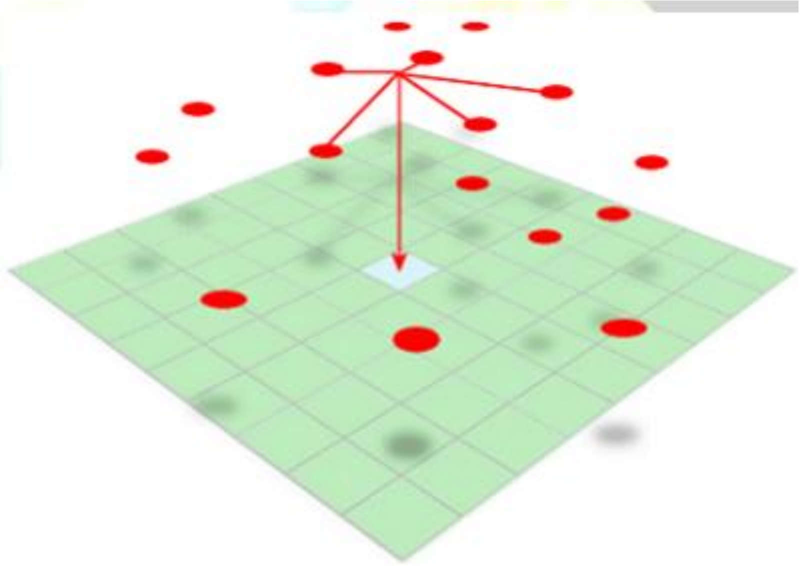
# What is spatial interpolation?

- Spatial interpolation calculates an unknown value from a set of sample points with known values that are distributed across an area.
- The distance from the cell with unknown value to the sample cells contributes to its final value estimation.



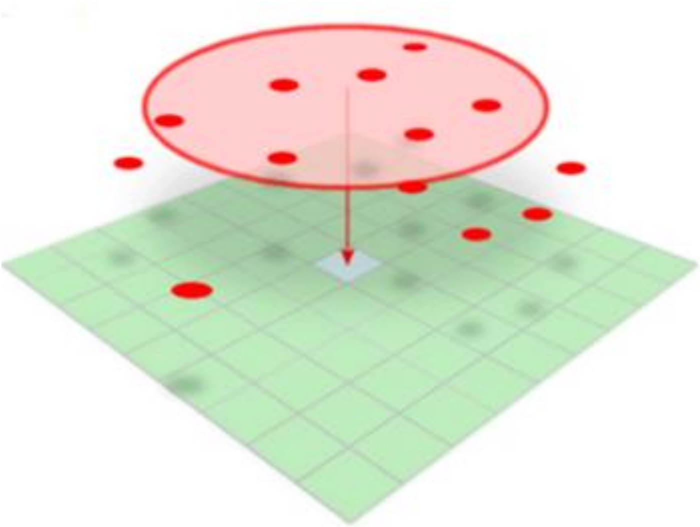
# Sample size

- Most interpolation methods allow you to control the number of sample points used to estimate cell values.
- For example, if you limit your sample to five points, the interpolator will use the five nearest points to estimate cell values.
- The distance to each sample point will vary depending on the distribution of the points.
- If you have a lot of sample points, reducing the size of the sample you use will speed up the interpolation process because a smaller set of numbers will be used to estimate each cell value.



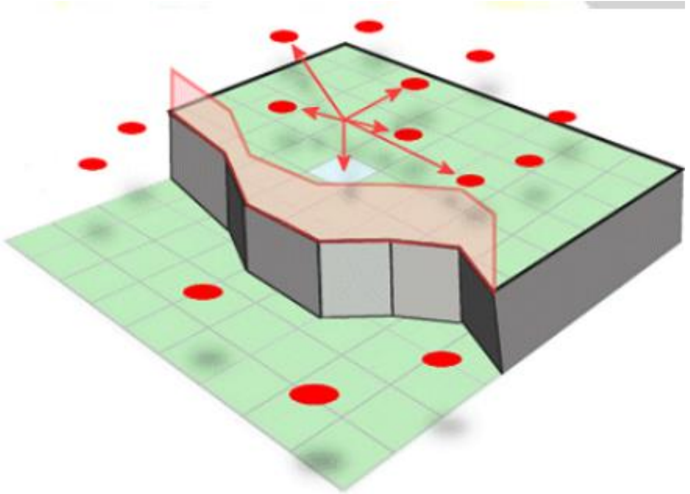
# Sample size radius

- You can also control your sample size by defining a search radius.
- The number of sample points found within a search radius can vary depending on how the points are distributed.
- You can choose to use some or all of the samples that fall within this radius to calculate the cell value.
- A variable search radius will continue to expand until the specified sample size is found.
- A fixed search radius will use only the samples contained within it, regardless of how many or how few that might be.



# Interpolation barriers

- Most interpolators attempt to smooth over these differences by incorporating and averaging values on both sides of the barrier.
- The Inverse Distance Weighted method allows you to include barriers in the analysis.
- The barrier prevents the interpolator from using samples points on one side of it.



When you use a barrier with interpolation, the estimated cell value is calculated from sample points on one side of the barrier

# INTERPOLATION METHODS

# Main interpolation methods

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- The Inverse Distance to a Power method
- The Kriging Method
- The Minimum Curvature Method
- The Modified Shepard's Method
- The Natural Neighbor Method
- The Nearest Neighbor Method
- The Polynomial Regression Method

# Other interpolation methods

- The Radial Basis Function Interpolation Method
- The Triangulation with Linear Interpolation Method
- The Moving Average Method
- The Data Metrics Methods
- The Local Polynomial Method

# The Inverse Distance to a Power method

# The Inverse Distance to a Power method

- The Inverse Distance to a Power method is a weighted average interpolator, which can be either exact or smoothing.
- With Inverse Distance to a Power, data are weighted during interpolation, so that the influence of one point, relative to another, declines with distance from the grid node.
- Weighting is assigned to data through the use of a weighting power, which controls how the weighting factors drop off as distance from the grid node increases.
- The greater the weighting power, the less effect the points, far removed from the grid node, have during interpolation.
- As the power increases, the grid node value approaches the value of the nearest point.
- For a smaller power, the weights are more evenly distributed among the neighboring data points. Normally, Inverse Distance to a Power behaves as an exact interpolator.

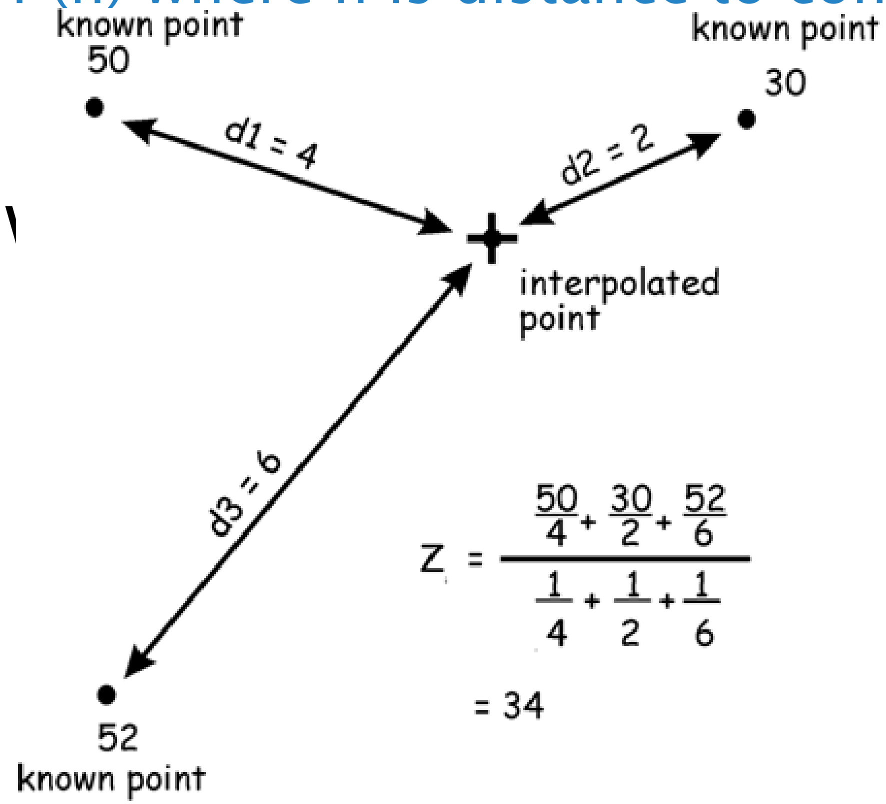
# Inverse Distance Weighted (IDW)



- Inverse Distance Weighted (IDW): z = f (h) where h is distance to control points

- As the distance increases, you

  values



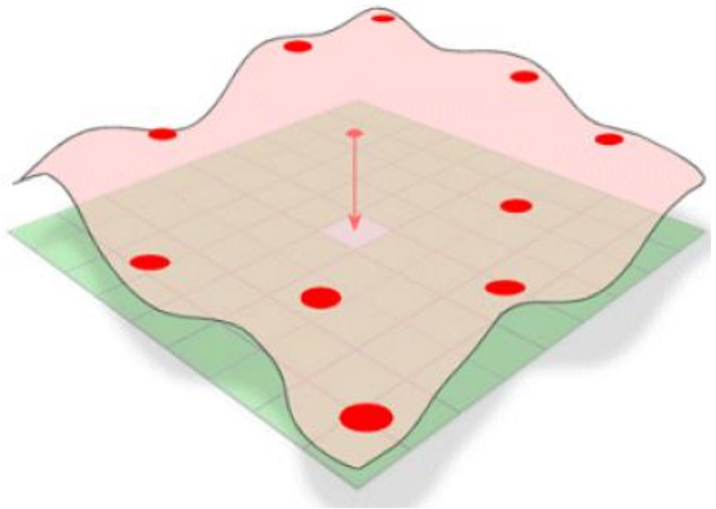
- When calculating a grid node, the weights assigned to the data points are fractions, the sum of all the weights being equal to 1.0.
- When a particular observation is coincident with a grid node, the distance between that observation and the grid node is 0.0, that observation is given a weight of 1.0; all other observations are given weights of 0.0. Thus, the grid node is assigned the value of the coincident observation.

- The smoothing parameter is a mechanism for buffering this behavior.
- When you assign a non-zero smoothing parameter, no point is given an overwhelming weight, meaning that no point is given a weighting factor equal to 1.0. One of the characteristics of Inverse Distance to a Power is the generation of "bull's-eyes" surrounding the observation position within the grid area.
- A smoothing parameter can be assigned during Inverse Distance to a Power to reduce the "bull's-eye" effect by smoothing the interpolated grid.

# Spline method

# Spline

- Spline virtually guarantees you a smooth-looking surface.
- Imagine stretching a rubber sheet so that it passes through all of your sample points



- Spline functions imitates a thin flexible sheet forced to pass close to the data points
- The equilibrium shape of the sheet minimizes the bending energy which is closely related to the surface curvature
- Repeatedly applies a smoothing equation (piecewise polyno

$$\sum_{j=1}^{N} |z_j - F(\mathbf{r}_j)|^2 w_j + w_0 I(F) = minimum$$

- –Resul

# Kriging method

# Kriging

- Kriging: z = f (h, v) + r where v is the semivariogram model, and r is the residual (i.e. difference between model and observed value)
- Similar to IDW in that
  - A grid is overlaid on top of control points, and the goal is to derive values at a grid point from control points
  - Values at a grid are determined by values at nearby control points weighted by inverse distance
- Different from IDW in that
  - It builds the model of spatial autocorrelation from known values (called "semivariogram"), and the weights are determined such that observed values are best fitted into the specified model
  - By model-fitting mechanism, the estimated values are supposed to reflect the spatial structure of given data; it also provides the way to validate the weights (e.g. standard error of the estimate)

# Types of Kriging

- Kriging methods can be classified as linear and non-linear methods. All nonlinear kriging algorithms are actually linear kriging applied to specific nonlinear transforms of the original data.

# Types of Kriging

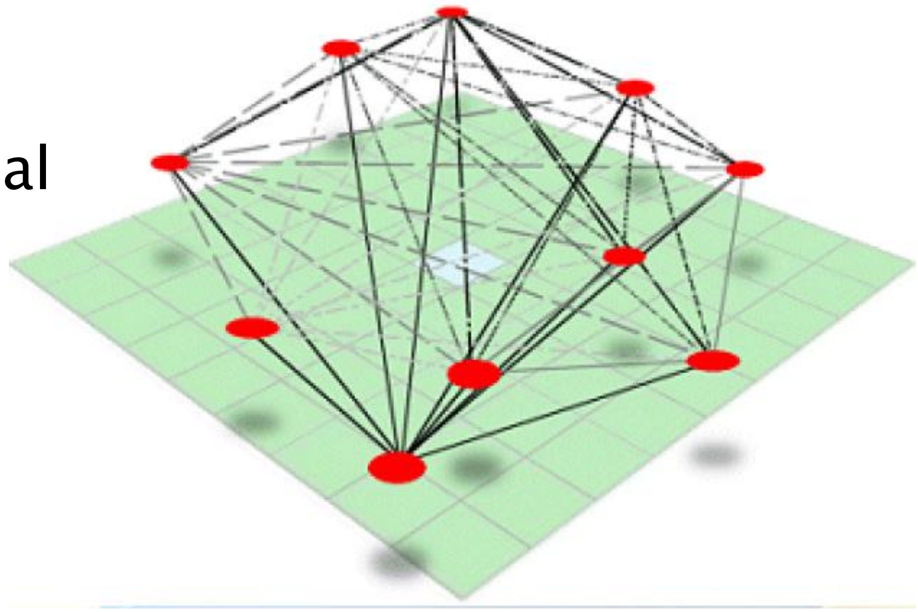


- Simple Kriging (SK)
- Ordinary Kriging (OK)
- Universal Kriging (UK)
- Disjunctive Kriging (DK)
- Indicator Kriging (IK)
- CoKriging (COK)
- Lognormal Kriging (LK)

- Simple, ordinary, and Universal Kriging predictors are all linear predictors, meaning that prediction at any location is obtained as a weighted average of neighboring data.

# Kriging

- Kriging is one of the most complex and powerful interpolators.
- It applies sophisticated statistical methods that consider the unique characteristics of your dataset. In order to use Kriging interpolation properly, you should have a solid understanding of geostatistical concepts and methods.



- Kriging is based on the idea that you can make inferences regarding a random function Z(x), given data points Z(x1), Z(x2), …Z(xn)
- The basis of this technique is the rate at which the variance between points changes over space
- This is expressed in the semivariogram which shows how the average difference between values at points changes with distance between points

# Numeric example of kriging

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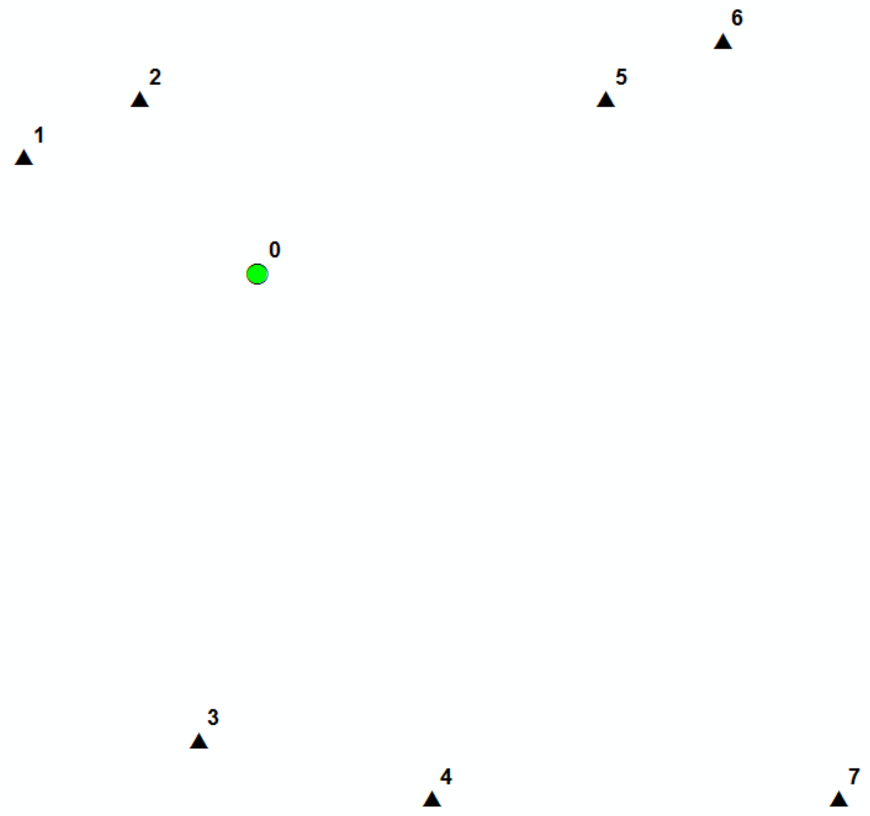
- In this example, we want to estimate a value for point 0 (65E, 137N), based on the 7 surrounding sample points.
- The table indicates the (x,y) coordinates of the 7 sample points, their corresponding values of Z (which is the variable we are interested in) and their distance to point 0.

| No | X | Y | Z | Dis From 0 |
|---|---|---|---|---|
| 0 | 65 | 137 | ???? | 0.000 |
| 1 | 61 | 139 | 477 | 4.472 |
| 2 | 63 | 140 | 696 | 3.606 |
| 3 | 64 | 129 | 227 | 8.062 |
| 4 | 68 | 128 | 646 | 9.487 |
| 5 | 71 | 140 | 606 | 6.708 |
| 6 | 73 | 141 | 791 | 8.944 |
| 7 | 75 | 128 | 783 | 13.454 |



# Solution this example With IDW

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| No | X | Y | Z | Dis From 0 | w=1/d^2 | z*W |
|---|---|---|---|---|---|---|
| 0 | 65 | 137 | | | | |
| 1 | 61 | 139 | 477 | 4.472 | 0.050003 | 23.85145 |
| 2 | 63 | 140 | 696 | 3.606 | 0.076904 | 53.52514 |
| 3 | 64 | 129 | 227 | 8.062 | 0.015386 | 3.492531 |
| 4 | 68 | 128 | 646 | 9.487 | 0.011111 | 7.177525 |
| 5 | 71 | 140 | 606 | 6.708 | 0.022224 | 13.46749 |
| 6 | 73 | 141 | 791 | 8.944 | 0.012501 | 9.888101 |
| 7 | 75 | 128 | 783 | 13.454 | 0.005525 | 4.325725 |
| | | | | total | 0.193652 | 115.728 |

Z= 115.7279559 / 0.19365218= 597.607

# Kriging matrices

$$\begin{array}{ccccccc} \mathbf{C} & \cdot & \mathbf{w} & = & \mathbf{D} \\ \underbrace{\begin{bmatrix} \tilde{C}_{11} & \cdots & \tilde{C}_{1n} & 1 \\ \vdots & \ddots & \vdots & \vdots \\ \tilde{C}_{n1} & \cdots & \tilde{C}_{nn} & 1 \\ 1 & \cdots & 1 & 0 \end{bmatrix}}_{(n+1)\times(n+1)} & \cdot & \underbrace{\begin{bmatrix} w_1 \\ \vdots \\ w_n \\ \mu \end{bmatrix}}_{(n+1)\times 1} & = & \underbrace{\begin{bmatrix} \tilde{C}_{10} \\ \vdots \\ \tilde{C}_{n0} \\ 1 \end{bmatrix}}_{(n+1)\times 1} \end{array}$$

$$\begin{aligned} \mathbf{C}\cdot\mathbf{w} &= \mathbf{D} \\ \mathbf{C}^{-1}\cdot\mathbf{C}\cdot\mathbf{w} &= \mathbf{C}^{-1}\cdot\mathbf{D} \\ \mathbf{I}\cdot\mathbf{w} &= \mathbf{C}^{-1}\cdot\mathbf{D} \\ \mathbf{w} &= \mathbf{C}^{-1}\cdot\mathbf{D} \end{aligned}$$

# Kriging matrices

- First, the distance matrix.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | 0.000 | 4.472 | 3.606 | 8.062 | 9.487 | 6.708 | 8.944 | 13.454 |
| 1 | 4.472 | 0.000 | 2.236 | 10.440 | 13.038 | 10.050 | 12.166 | 17.804 |
| 2 | 3.606 | 2.236 | 0.000 | 11.045 | 13.000 | 8.000 | 10.050 | 16.971 |
| 3 | 8.062 | 10.440 | 11.045 | 0.000 | 4.123 | 13.038 | 15.000 | 11.045 |
| 4 | 9.487 | 13.038 | 13.000 | 4.123 | 0.000 | 12.369 | 13.928 | 7.000 |
| 5 | 6.708 | 10.050 | 8.000 | 13.038 | 12.369 | 0.000 | 2.236 | 12.649 |
| 6 | 8.944 | 12.166 | 10.050 | 15.000 | 13.928 | 2.236 | 0.000 | 13.153 |
| 7 | 13.454 | 17.804 | 16.971 | 11.045 | 7.000 | 12.649 | 13.153 | 0.000 |

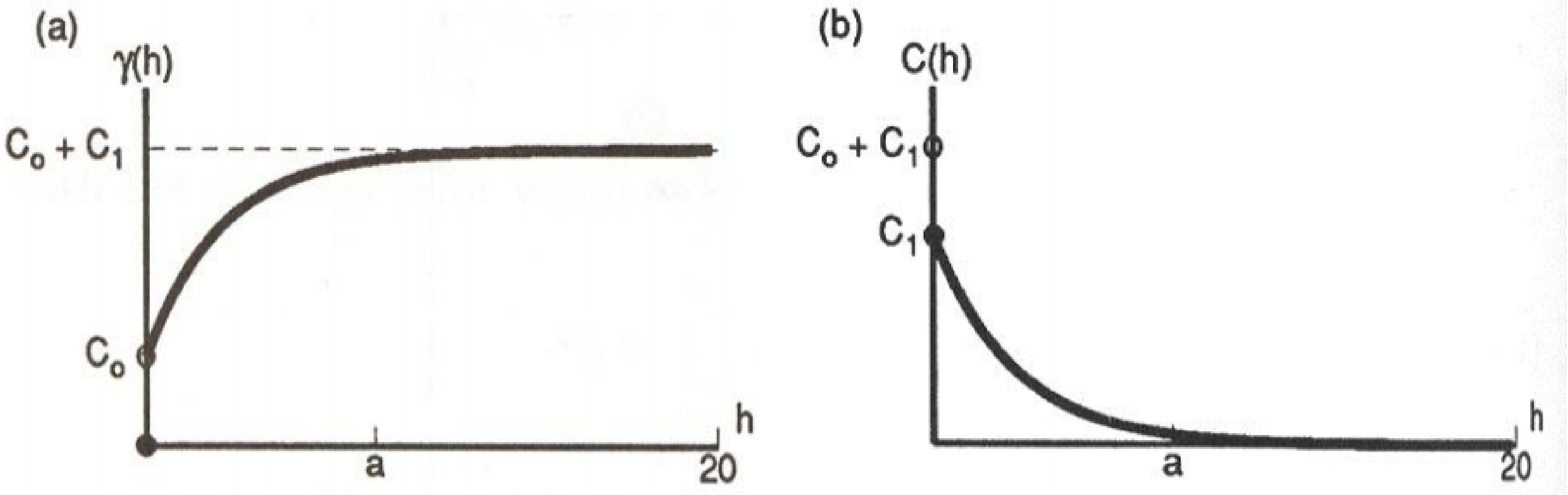
- variances will be calculated based on the distance between points using Exponential model :

Parameters:

$C_0 = 0$, $a = 10$, $C_1 = 10$

Variogram model

Covariance function




$$\gamma(h) = \begin{cases} 0 & \text{if } |h| = 0 \\ C_0 + C_1(\exp(\frac{-3|h|}{a})) & \text{if } |h| > 0 \end{cases}$$

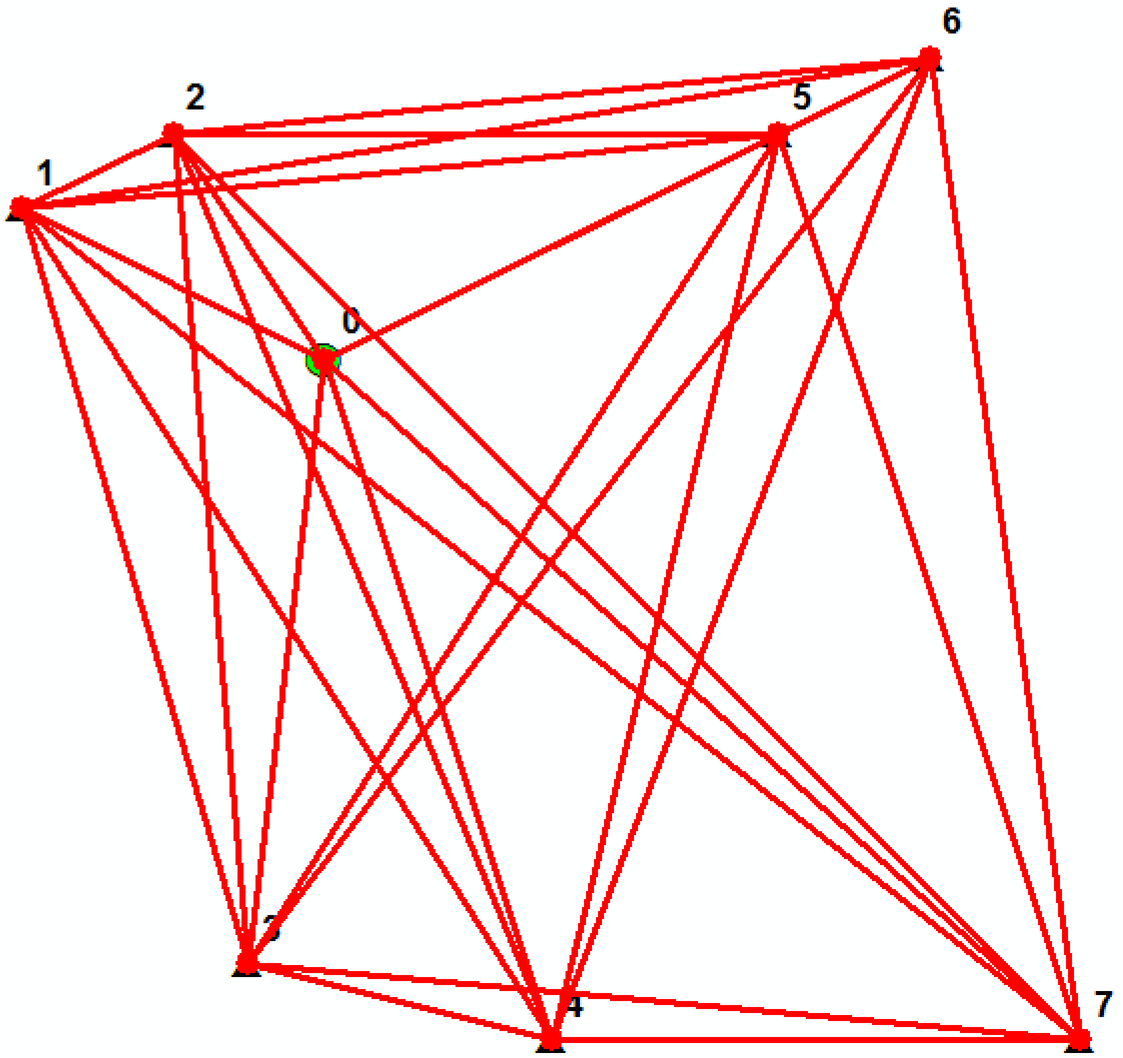
$$C(h) = \begin{cases} C_0 + C_1 & \text{if } |h| = 0 \\ C_1(\exp(\frac{-3|h|}{a})) & \text{if } |h| > 0 \end{cases}$$

$$\gamma(h) = c_0 + c_1\left(1 - e^{-h/\alpha}\right)$$

- where c0 is the nugget effect.

$$\gamma(h) = C(0) - C(h) \qquad C(h) = 10\, e^{-0.3|h|}$$

- The sill is c0+c1.
- The range for the exponential model is defined to be 3a at which the variogram is of 95% of the sill


6
2
5
1
0
3
4
7

# Kriging matrices

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 10.000 | 5.113 | 0.436 | 0.200 | 0.490 | 0.260 | 1.000 | 1 |
| 5.113 | 10.000 | 0.364 | 0.202 | 0.907 | 0.490 | 0.062 | 1 |
| 0.436 | 0.364 | 10.000 | 2.903 | 0.200 | 0.111 | 0.364 | 1 |
| 0.200 | 0.202 | 2.903 | 10.000 | 0.245 | 0.153 | 1.225 | 1 |
| 0.490 | 0.907 | 0.200 | 0.245 | 10.000 | 5.113 | 0.225 | 1 |
| 0.260 | 0.490 | 0.111 | 0.153 | 5.113 | 10.000 | 0.193 | 1 |
| 0.048 | 0.062 | 0.364 | 1.225 | 0.225 | 0.193 | 10.000 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

$$\mathbf{C} = \begin{bmatrix} \tilde{C}_{11} & \tilde{C}_{12} & \tilde{C}_{13} & \tilde{C}_{14} & \tilde{C}_{15} & \tilde{C}_{16} & \tilde{C}_{17} & 1 \\ \tilde{C}_{21} & \tilde{C}_{22} & \tilde{C}_{23} & \tilde{C}_{24} & \tilde{C}_{25} & \tilde{C}_{26} & \tilde{C}_{27} & 1 \\ \tilde{C}_{31} & \tilde{C}_{32} & \tilde{C}_{33} & \tilde{C}_{34} & \tilde{C}_{35} & \tilde{C}_{36} & \tilde{C}_{37} & 1 \\ \tilde{C}_{41} & \tilde{C}_{42} & \tilde{C}_{43} & \tilde{C}_{44} & \tilde{C}_{45} & \tilde{C}_{46} & \tilde{C}_{47} & 1 \\ \tilde{C}_{51} & \tilde{C}_{52} & \tilde{C}_{53} & \tilde{C}_{54} & \tilde{C}_{55} & \tilde{C}_{56} & \tilde{C}_{57} & 1 \\ \tilde{C}_{61} & \tilde{C}_{62} & \tilde{C}_{63} & \tilde{C}_{64} & \tilde{C}_{65} & \tilde{C}_{66} & \tilde{C}_{67} & 1 \\ \tilde{C}_{71} & \tilde{C}_{72} & \tilde{C}_{73} & \tilde{C}_{74} & \tilde{C}_{75} & \tilde{C}_{76} & \tilde{C}_{77} & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

The Inverse of C is:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0.128 | −0.075 | −0.011 | −0.006 | −0.006 | −0.007 | −0.022 | 0.113 |
| −0.078 | 0.129 | −0.011 | −0.010 | −0.016 | −0.009 | −0.004 | 0.135 |
| −0.013 | −0.010 | 0.098 | −0.042 | −0.010 | −0.010 | −0.013 | 0.159 |
| −0.009 | −0.009 | −0.042 | 0.102 | −0.009 | −0.009 | −0.024 | 0.141 |
| −0.008 | −0.015 | −0.010 | −0.009 | 0.130 | −0.077 | −0.011 | 0.119 |
| −0.009 | −0.008 | −0.010 | −0.009 | −0.077 | 0.126 | −0.012 | 0.143 |
| −0.012 | −0.011 | −0.014 | −0.025 | −0.012 | −0.013 | 0.086 | 0.191 |
| 0.138 | 0.123 | 0.158 | 0.143 | 0.119 | 0.143 | 0.177 | −2.205 |

$$\mathbf{D} = \begin{bmatrix} \tilde{C}_{10} \\ \tilde{C}_{20} \\ \tilde{C}_{30} \\ \tilde{C}_{40} \\ \tilde{C}_{50} \\ \tilde{C}_{60} \\ \tilde{C}_{70} \\ 1 \end{bmatrix}$$

| |
|---|
| 2.614 |
| 3.390 |
| 0.890 |
| 0.581 |
| 1.337 |
| 0.683 |
| 0.177 |
| 1 |

| |
|---|
| 0.162 |
| 0.324 |
| 0.130 |
| 0.087 |
| 0.152 |
| 0.058 |
| 0.087 |
| −0.918 |

# Kriging weights:

$$\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \\ \mu \end{bmatrix} = \mathbf{C}^{-1} \cdot \mathbf{D} =$$

- Estimated value for point O:
- (477)( 0.162)+ (696)( 0.324)+( 227)( 0.130)+( 646)( 0.087)+( 606)( 0.152)+( 791)( 0.058)+( 783)( 0.087) = 594.5796